Infinite Positive Semidefinite Tensor Factorization for Source Separation of Mixture Signals

Kazuyoshi Yoshii\textsuperscript{1}  Ryota Tomioka\textsuperscript{2}  Daichi Mochihashi\textsuperscript{3}  Masataka Goto\textsuperscript{1}
\textsuperscript{1}National Institute of Advanced Industrial Science and Technology (AIST)
\textsuperscript{2}The University of Tokyo  \textsuperscript{3}The Institute of Statistical Mathematics (ISM)

### Conventional: Nonnegative Matrix Factorization (NMF)
Each nonnegative vector is approximated by a convex combination of nonnegative vectors.

\[ x_n \approx \sum_{k=1}^{K} \omega_k h_{kn} \]

- **Vector-wise factorization**
- **Basis matrix** \( X = [x_1, \ldots, x_N] \in \mathbb{R}^{M \times N} \)
- **Activation matrix** \( H = [h_1, \ldots, h_K] \in \mathbb{R}^{K \times N} \)
- **Reconstruction error** (Bregman divergence)
  \[ \phi(x) = \| \phi(x) - \phi(y_n) \| \]
  \[ D_k(x_n, y_n) = \phi(x_n) - \phi(y_n) + \phi(y_n)^T(x_n - y_n) \]
- **Kullback-Leibler (KL) divergence**
  \[ D_k(x_n, y_n) = \sum_m \left( x_{mn} \log x_{mn} - x_{mn} - y_{mn} \log y_{mn} + y_{mn} \right) \]
- **Itakura-Saito (IS) divergence**
  \[ D_k(x_n, y_n) = \sum_m \left( \log x_{mn} y_{mn} - x_{mn} - y_{mn} \right) \]

Given \( X \), NMF tries to estimate \( W \) and \( H \) such that \( C(X|Y) = \sum_n D_k(x_n, y_n) \) is minimized.

- **Maximum likelihood estimation**

### Proposed: Positive Semidefinite Tensor Factorization (PSDTF)
Each positive semidefinite matrix is approximated by a convex combination of positive semidefinite matrices.

\[ X_n \approx \sum_{k=1}^{K} V_k h_{kn} \]

- **Matrix-wise factorization**
- **Observed tensor** (PSD matrices)
  \( X = [X_1, \ldots, X_N] \in \mathbb{R}^{M \times N \times N} \)
- **Basis tensor** (PSD matrices)
  \( V = [V_1, \ldots, V_K] \in \mathbb{R}^{M \times K} \)
- **Activation (nonnegative tensors)**
  \( H = [h_1, \ldots, h_K] \in \mathbb{R}^{K \times N} \)

Reconstruction error (Bregman divergence)
\[ D_k(X_n|Y_n) = \phi(X_n) - \phi(Y_n) - \text{tr}(V_f \phi(Y_n)^T(X_n - Y_n)) \]

von Neumann (vN) divergence
\[ D_k(X_n|Y_n) = \text{tr}(X_n \log X_n - X_n + Y_n) \]

Log-determinant (LD) divergence
\[ D_k(X_n|Y_n) = -\log |X_n| + \text{tr}(X_n^{-1} - Y_n) \]

- **Global weight vector** \( \theta = \left[ \theta_1, \ldots, \theta_K \right] \in \mathbb{R}^K \)

### Application to Single-Channel Audio Signal Separation
- **Time-domain decomposition** of mixture signals based on LD-PSDTF and Wiener filtering.

\[ \hat{x}_n \sim N(0, V_k) \]

The reproducing property of the Gaussian gives
\[ \log p(X_n|Y_n) \leq -\frac{1}{2} \log |Y_n| - \frac{1}{2} \text{tr}(X_n^{-1}) \]

### Application to Multi-Channel EEG Signal Analysis
- **Unsupervised learning** of characteristic brain activity patterns based on LD-PSDTF.

Task: predict a left or right hand movement (-1 or 1) for a given EEG.
- **Future Work**
  - Reduce the heavy computational cost
  - Investigate vN-PSDTF (extension of KL-NMF)